

SM3 1.2: Pascal's Triangle & Binomial Thm Key

Problems: Expand the binomial.

1) $(a + b)^3$

$$\binom{3}{0}(a)^3(b)^0 + \binom{3}{1}(a)^2(b)^1 + \binom{3}{2}(a)^1(b)^2 + \binom{3}{3}(a)^0(b)^3$$

$$(1)(a^3)(1) + (3)(a^2)(b) + (3)(a)(b^2) + (1)(1)(b^3)$$

$$a^3 + 3a^2b + 3ab^2 + b^3$$

2) $(a + b)^5$

$$\binom{5}{0}(a)^5(b)^0 + \binom{5}{1}(a)^4(b)^1 + \binom{5}{2}(a)^3(b)^2 + \binom{5}{3}(a)^2(b)^3 + \binom{5}{4}(a)^1(b)^4 + \binom{5}{5}(a)^0(b)^5$$

$$(1)(a^5)(1) + (5)(a^4)(b) + (10)(a^3)(b^2) + (10)(a^2)(b^3) + (5)(a)(b^4) + (10)(1)(b^5)$$

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

3) $(a + 3)^4$

$$\binom{4}{0}(a)^4(3)^0 + \binom{4}{1}(a)^3(3)^1 + \binom{4}{2}(a)^2(3)^2 + \binom{4}{3}(a)^1(3)^3 + \binom{4}{4}(a)^0(3)^4$$

$$(1)(a^4)(1) + (4)(a^3)(3) + (6)(a^2)(9) + (4)(a)(27) + (1)(1)(81)$$

$$a^4 + 12a^3 + 54a^2 + 108a + 81$$

4) $(x - 2)^6$

$$\binom{6}{0}(x)^6(-2)^0 + \binom{6}{1}(x)^5(-2)^1 + \binom{6}{2}(x)^4(-2)^2 + \binom{6}{3}(x)^3(-2)^3 + \binom{6}{4}(x)^2(-2)^4 + \binom{6}{5}(x)^1(-2)^5$$

$$+ \binom{6}{6}(x)^0(-2)^6$$

$$(1)(x^6)(1) + (6)(x^5)(-2) + (15)(x^4)(4) + (20)(x^3)(-8) + (15)(x^2)(16) + (6)(x)(-32)$$

$$+ (1)(1)(64)$$

$$x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$$

5) $(2x - 1)^5$

$$\binom{5}{0}(2x)^5(-1)^0 + \binom{5}{1}(2x)^4(-1)^1 + \binom{5}{2}(2x)^3(-1)^2 + \binom{5}{3}(2x)^2(-1)^3 + \binom{5}{4}(2x)^1(-1)^4$$

$$+ \binom{5}{5}(2x)^0(-1)^5$$

$$(1)(32x^5)(1) + (5)(16x^4)(-1) + (10)(8x^3)(1) + (10)(4x^2)(-1) + (5)(2x)(1)$$

$$+ (1)(1)(-1)$$

$$32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$$

$$\begin{aligned}
 6) \quad & (x + 5y)^3 \\
 & \binom{3}{0}(x)^3(5y)^0 + \binom{3}{1}(x)^2(5y)^1 + \binom{3}{2}(x)^1(5y)^2 + \binom{3}{3}(x)^0(5y)^3 \\
 & (1)(x^3)(1) + (3)(x^2)(5y) + (3)(x)(25y^2) + (1)(1)(125y^3) \\
 & x^3 + 15x^2y + 75xy^2 + 125y^3
 \end{aligned}$$

Find the given term in the binomial expansion:

$$\begin{aligned}
 7) \quad & x^6 \text{ term; } (x - 1)^9 \\
 & \binom{9}{6}(x)^6(-1)^{9-6} \\
 & \binom{9}{6}(x)^6(-1)^3 \\
 & (84)(x^6)(-1) \\
 & -84x^6
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & x^6y^3 \text{ term; } (x + y)^9 \\
 & \binom{9}{6}(x)^6(y)^{9-6} \\
 & \binom{9}{6}(x)^6(y)^3 \\
 & (84)(x^6)(y^3) \\
 & 84x^6y^3
 \end{aligned}$$

$$\begin{aligned}
 8) \quad & x^4 \text{ term; } (2x + 3)^7 \\
 & \binom{7}{4}(2x)^4(3)^{7-4} \\
 & \binom{7}{4}(2x)^4(3)^3 \\
 & (35)(16x^4)(27) \\
 & 15120x^4
 \end{aligned}$$

$$\begin{aligned}
 10) \quad & x^{11}y^2 \text{ term; } (x - 2y)^{13} \\
 & \binom{13}{11}(x)^{11}(-2y)^{13-11} \\
 & \binom{13}{11}(x)^{11}(-2y)^2 \\
 & (78)(x^{11})(4y^2) \\
 & 312x^{11}y^2
 \end{aligned}$$